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THE ORDER OF TEACHING THE PARTS OF THE CALCULUS.

By John K. Lamond.

When the average student reaches the calculus his ideas of a function, a variable, a variable approaching a limit, etc., are so vague, and oftentimes so entirely wrong, that it would seem wise, since the foundations of the calculus rest so largely on the theory of limits, to spend enough time at the beginning of the course to develop the theory of limits in a careful and thorough manner.

It is the present tendency to mix the differential and integral calculus. That is, to develop the two divisions side by side. Theoretically this may seem like an ideal thing to do, for the student will be made to see the interrelation of the two divisions of the subject from the very start. But in practice, since the ideas of the calculus are so new to the student, and so very much bigger than anything which he has encountered during his previous mathematical experience, it seems doubtful if the *average student* gains anything from such a treatment. Certainly he loses nothing, if the formulas for differentiating are developed first, with no mention made of the inverse operation, and the subject is not so likely to become confusing to the poorer students.

Having developed these formulas, the thing that the student is interested in is not another series of inverse operations. The thing he is continually asking himself and his instructor is, "What is this derivative, this thing we have spent several weeks in learning how to find, really good for now that we have it?" This natural curiosity is amply satisfied by maxima and minima, and rates. Having finished these subjects, if the other courses which the student is taking are such that some knowledge of integral calculus seems desirable, the indefinite integral, or both the indefinite and definite integral, may be very profitably introduced at this time, leaving the remaining topics of the differential calculus for later treatment.

From this point it seems doubtful if one can say just what is best. Each individual teacher, knowing the capabilities and needs of his class, is the best judge of what to omit, and what to give, and the order of the giving.

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BY C. C. GROVE.

The purpose of the committee in setting this topic is not quite clear. All that I can say, it seems, must have occurred to every one that has taught the subject. Some of the orders of presenting any school subject may be termed the historical order, the logical order, and the psychological order.

In the case of the calculus, the historical order is twofold according to the point of view or of the time and place of beginning. Despite this dilemma for us, there is much to be said in favor of an historical order of presenting a subject. It seems to fit into the student's growing capacities. He likes to see any living thing develop. In mathematics, however, it would sometimes be quite at variance with the logical order into which we so naturally go the more a subject becomes crystallized in our minds.

Last evening a friend was speaking of the piano and organ as means of musical expression. He felt the piano superior because its impulses of tone, its discontinuous notes, are suggestive and excite the imagination; whereas, the sustained notes of the organ melt into one grand finished product that we quietly admire. It seems to me that the logical order is rather like the organ, and that it is quite easy to have a presentation so logically coherent, and so naturally and simply and clearly knit together that the student feels nothing remains to be said or done, and yet becomes hopelessly lost when he tries to reproduce that "simple" presentation. The student's mind too needs to traverse some, at least, of that devious path by which the instructor reached his present logical formulation.

The third order combines the former two with the consciousness of that living interaction between instructor and class that enables the topics to be brought up at the pedagogically critical moment. This will vary somewhat with every combination of teacher and class. Thus it is, in part, that so many different textbooks appear. The teacher must direct this procedure carefully and good results are likely to follow.

As to the differential and the integral parts of the calculus, several years of teaching according to each of five different texts have led me to prefer to develop all the differential formulas before introducing integration, as the reverse process and then as a summation. We find no difficulty in maintaining interest through the more formal part, or any part for that matter. There is a growing interest as new power is gained to attack problems. The development of the subject is continually reviewing the mathematics studied before and bringing up new applications of that former material that furnish true delight to the student.

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BY ROSS W. MARRIOTT.

The subject of to-day's discussion is one which demands the close consideration of all teachers of the calculus, and to my mind resolves itself into the question as to whether we should hold to the time-honored custom of the presentation of the calculus or whether we should fit it to the needs of the present-day student. It seems to me, for example, that to take up the whole of the differential processes without considering the process inverse, is as unnatural as it would be in arithmetic or algebra to treat multiplication of all types of numbers before considering the operation of division, which I believe is seldom if ever done. Just as we sometimes treat division as a multiplication process, so we have occasion at times to treat integration as a process of differentiation. The processes of differentiation and its inverse are so closely allied that I believe there is an advantage gained by studying the elementary standard forms of the integration at the time we study the differential forms. Such modes of integration as require a transformation process could well be left to study under the integral calculus proper.

I think the order in which the calculus was invented has had a great deal to do with the manner in which it has been presented. When the differential calculus was invented it was found that the inverse process gave results identical with the older integral calculus, which depended in no way upon the differential, hence the formal division into the two branches of calculus.

The demand for the calculus is continually growing, and comes from a varied class of students. Such subjects as engineering, chemistry, economics, and biology all have a claim upon calculus as an auxiliary, but many of the students of these subjects cannot afford to give time to an extended study of calculus.

If, then, we find that the calculus by a different mode of presentation better meets the needs of our students, we are justified in making such a presentation of its parts.

The arrangement of the parts of the calculus taught depends upon the object or end for which the student takes the calculus. It must be so selected that the work does not degenerate into mere mechanical routine, while at the same time the student becomes well grounded in the formal processes which are so necessary for the intelligent application of any branch of mathematics.

Simple practical application of the elementary portions should be introduced very early, the geometrical and physical significance of the derivative as soon as it is defined, and problems relating to it, may be introduced. Some of the elementary portions of curve tracing, maximum and minimum and rates can be taken up with advantage along with the formal processes of differentiation.

However, the practicality may be over done in the early stages of the calculus, and the student may lose sight of the significance of the formal processes, and so never be able to make much use of the calculus, as the real applications require a thorough notion of the formal processes.

A certain well-accepted textbook on calculus gives the symbol for an inverse circular function, and then states in italics that it is the angle whose sine is so and so. Shortly afterwards it makes an application in which this function enters additively with a pure scalar number. A student not long ago asked how it were possible to add them and get the measure of an area. This is what I call too much practicality or rather too much without sufficient preliminary formality.

I do not think such subjects as Taylor's theorem, theorem of mean, expansion of functions, etc., which are primarily applications of the differential calculus should be left until all the formal processes of integration are completed, as is done in so many textbooks, for these things have as wide an application as the calculus proper, and are indeed necessary for a logical and rigorous development of the calculus.

The significance of the calculus, then, the possibility of applying it in other fields, in short, its usefulness as an instrument should be kept constantly before the student during the study of the subject rather than deferred to some indefinite future.

A well-known educator has said that there is no more vicious educational practice, nor scarcely any more common one, than that of keeping the student in the dark as to the end and purpose of his work, for it breeds indifference and despair. The significance and usefulness of the calculus should not be kept from the student by following a time-honored custom, as the mysteries of a secret society are kept from the initiate until he has mastered the preceding degrees.

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